Seminar Handout (Week 6,7)

Linear Symmetric Hyperbolic Systems

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1. Definitions

$$L := A^0\partial_t + \sum_{j=1}^{n} A^j\partial_j + B$$

$$a_0 := \min_{v,t,x;|v|=1} A^0(t,x)v\cdot\overline{v} > 0$$

$$a_1 := \max_{v,t,x,j;|v|=1} A^j(t,x)v\cdot\overline{v} > 0$$

$$\alpha := a_0/(na_1)$$

$$K^\alpha := \{(t,x)|x\in K_t, 0\leqslant t\leqslant t_0\}$$

$$K_t := B(0,\frac{T_0-t}{\alpha})$$

$$|u(t)|_{K_t} := \left(\int_{K_t} A^0(t,x)u(t,x)\cdot\overline{u(t,x)}dx\right)^{\frac{1}{2}}$$

$$|u(t)|_{s,K_t} := \left(\sum_{|\alpha|\leqslant s}|\nabla^\alpha u(t)|^2_{K_t}\right)^{\frac{1}{2}}$$

2. Theorems

Lemma 0. There exists $a_0' > 0$ for any $t\in[0,T_0]$:

$$a_0||u(t)||^2_{L^2(K_t)} \leqslant |u(t)|^2_{K_t} \leqslant a_0'||u(t)||^2_{L^2(K_t)}$$

Corollary 3.2. Let $s\in\mathbb{N}$, $A^0,A^1,\cdots,A^n,B\in C^s_b$ and let $u\in C^{s+1}(K^\alpha(T_0))$ be a solution to

$$Lu = f\in C^s(K^\alpha(T_0)),\ \ u(t=0)=u_0\in C^s(K_0).$$

Then

$$\exists c = c(||(A^0,A^1,\cdots,A^n,B)||_{C^s(K^\alpha(T_0))}),\ \ \forall t\in[0,T_0]:$$

$$|u(t)|_{s,K_t} \leqslant c\left[|u_0|_{s,K_0} + \left(\int_0^{T_0}|f(r)|^2_{s,K_r}dr\right)^{\frac{1}{2}}\right]e^{ct}.$$

Corollary 3.2'. Or

$$||u(t)||_{W^{s,2}(K_t)} \leqslant c \left[||u_0||_{W^{s,2}(K_0)} + \left(\int_0^{T_0} ||f(r)||^2_{W^{s,2}(K_r)} dr \right)^{\frac{1}{2}} \right] e^{ct}.$$

Theorem 3.3. Let $s \in \mathbb{N},\ s > n/2+1,\ A^0, \cdots, A^n, B \in C_b^{s+1}[C_b^\infty]$ and let $u_0 \in W^{s,2}[\cap C^\infty]$. Then there exists unique solution $u \in C^1([0,\infty) \times \mathbb{R}^n)[C^\infty([0,\infty) \times \mathbb{R}^n)]$ to $Lu = 0,\ u(t = 0) = u_0$. Moreover

$$u \in C^0([0,\infty), W^{s,2}) \cap C^1([0,\infty), W^{s-1,2}).$$

References.

[1] R. Racke, *Lectures on Nonlinear Evolution Equations.*