Seminar Handout (Week 8)

Some Inequalities

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1. Definitions

Define $j : \mathbb{R}^n \to \mathbb{R}$ be a function satisfying

$$j \in C_0^\infty,\ j(x) = 0(|x| \geqslant 1),\ j \geqslant 0,\ \int_{\mathbb{R}^n} j(x)dx = 1.$$

Define **Friedrich mollifier** j_ϵ as

$$j_\epsilon(x) := \epsilon^{-n} j(\frac{x}{\epsilon}) \text{ for } \forall \epsilon > 0,\ x \in \mathbb{R}^n.$$

Define J_ϵ as

$$(J_\epsilon u)(x) := \int_{\mathbb{R}^n} j_\epsilon(x-y)u(y)dy =: (j_\epsilon * u)(x)$$

for $u \in L^1_{loc}(\mathbb{R}^n)$.

2. Lemmas

Lemma 4.2. Let $1 \leqslant p < \infty$ and let $u \in L^p$. Then

(i) $J_\epsilon u \in C^\infty$

(ii) $||J_\epsilon u||_p \leqslant ||u||_p$

(iii) $\lim_{\epsilon \downarrow 0} ||J_\epsilon u - u||_p = 0$

(iv) $\forall m \in \mathbb{N}_0,\ \forall q \geqslant p\ :\ J_\epsilon u \in W^{m,q}$.

Lemma 4.3. For any $\epsilon_0 > 0$, there exists $c > 0$ such that for any $\epsilon \in (0, \epsilon_0]$ and $u \in W^{1,2}$,

$$||J_\epsilon u - u||_2 \leqslant c\epsilon ||u||_{1,2}.$$

References.

[1] R. Racke, *Lectures on Nonlinear Evolution Equations.*