Seminar Handout (Week 5)

Linear Symmetric Hyperbolic Systems

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1. Definitions

$$|u(t)|_{K_t} := \left(\int_{K_t} A^0(t,x) u(t,x) \cdot \overline{u(t,x)} dx \right)^{\frac{1}{2}}$$

$$|u(t)|_{s,K_t} := \left(\sum_{|\alpha| \leqslant s} |\nabla^\alpha u(t)|^2_{K_t} \right)^{\frac{1}{2}}$$

2. Theorems

Theorem 3.1. Let $K^\alpha = K^\alpha(t_0)$ and let $u \in C^1(K^\alpha(T_0))$ be a solution to

$$Lu = f \in C^0(K^\alpha(T_0)), \;\; u(t=0) = u_0 \in C^0(K_0).$$

Then

$$\exists c = c(||(A^0, \partial_t A^0, \partial_1 A^1, \cdots, \partial_n A^n, B)||_{C^0(K^\alpha(T_0))}), \;\; \forall t \in [0, T_0] :$$

$$|u(t)|_{K_t} \leqslant c \left[|u_0|_{K_0} + \left(\int_0^{T_0} |f(r)|^2_{K_r} dr \right)^{\frac{1}{2}} \right] e^{ct}.$$

Corollary 3.2. Let $s \in \mathbb{N}$, $A^0, A^1, \cdots, A^n, B \in C^s_b$ and let $u \in C^{s+1}(K^\alpha(T_0))$ be a solution to

$$Lu = f \in C^s(K^\alpha(T_0)), \;\; u(t=0) = u_0 \in C^s(K_0).$$

Then

$$\exists c = c(||(A^0, A^1, \cdots, A^n, B)||_{C^s(K^\alpha(T_0))}), \;\; \forall t \in [0, T_0] :$$

$$|u(t)|_{s,K_t} \leqslant c \left[|u_0|_{s,K_0} + \left(\int_0^{T_0} |f(r)|^2_{s,K_r} dr \right)^{\frac{1}{2}} \right] e^{ct}.$$

References.

[1] R. Racke, *Lectures on Nonlinear Evolution Equations.*